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LETTER TO THE EDITOR

Linear screening by a 2DEG in the presence of a low magnetic field

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Abstract. A study is made of the linear screening response of a 2DEG in the presence of a low perpendicular magnetic field. The response is calculated by solving the linearized equation of motion for the density operator. Use is made of a Wigner-type representation which avoids the need to evaluate matrix elements between Landau wavefunctions. We give an explicit expression for the response to a static applied potential at zero temperature. The correction to the zero-magnetic-field response is significant for fields as low as 0.03 T. These corrections could therefore be important for the low-field magnetoresistance of systems such as dot and antidot lattices.

In recent years there has been growing theoretical interest in the screening response of a two-dimensional electron gas (2DEG) to an applied electrostatic field [1–9]. Much of the motivation for this has been from the study of dot and antidot lattices. These can be created in many ways [10–13], but most techniques ultimately involve the creation of a periodic electrostatic potential which is felt by the 2DEG and modulates its density. It is important to know the shape of the resulting self-consistent screened potential in which the electrons will reside. For instance, many successful theoretical treatments of transport in antidot lattices involve considering the classical motion of electrons in model screened potentials [14, 15]. The shape of the actual potential may then be important as it affects the electron trajectories.

Up to the present, calculations of the screening response of a 2DEG have concentrated on the simple cases of zero [7–9] or strong [2, 1, 16, 17, 3, 5, 6] magnetic field. In this letter we shall consider the more difficult case of response in the presence of a low or moderate magnetic field.

We will build upon much work performed to determine the response of a three-dimensional electron gas. This has been studied using a variety of one-particle, many-body and diagrammatic techniques. Some of the earliest work was by Sondheimer and Wilson [18] who considered the partition function of free electrons in a magnetic field, followed by Nozières and Pines [19] who introduced a generalized dielectric constant for a many-body problem. Soon after this, much work was performed by Stephen [20, 21] using Green function techniques, and Quinn and Rodriguez [22] who calculated the conductivity in a magnetic field. A very important paper by Hohenberg and Kohn [23] introduced the use of density functional techniques and studied in detail the Thomas Fermi approximation. Some very nice work was also done by Mermin *et al* [24, 25] who used Hartree–Fock and random phase approximations. Our calculation is greatly simplified by use of a representation first given by Stinchcombe [26]. This removes the need to evaluate matrix elements between Landau states which have a pathological low-field limit.

The development consists of two main sections. In the first, the linearized equation of motion for the single-particle density operator will be rewritten using the representation of [26], thus reducing it to a form where a solution can be found. The second section then studies the static response. The formulation for this section will closely follow that used in [24]. We shall use a *single-particle picture* which will avoid complications due to keeping antisymmetry under exchange of fermions.

Let ρ be the single-particle density operator defined by

$$\rho = \sum_{i,\sigma} |i, \sigma\rangle\, f(\epsilon_{i,\sigma})\, \langle i, \sigma| \tag{1}$$

where $f(\epsilon)$ is the Fermi–Dirac distribution function, and $|i, \sigma\rangle$ are the eigenstates of the system with Hamiltonian H. σ represents spin variables. The electron density is then given by $n(\boldsymbol{r}) = \sum_\sigma \langle \boldsymbol{r}|\rho|\boldsymbol{r}\rangle$. We will split the single-particle Hamiltonian into two parts, $H = H_0 + H_1$, where H_0 is the Hamiltonian for an electron confined to a 2D plane normal to $\hat{\boldsymbol{z}}$ in the presence of a magnetic field $\boldsymbol{B} = B\hat{\boldsymbol{z}}$,

$$H_0 = \frac{1}{2m}\,(\boldsymbol{p} + |e|\boldsymbol{A})^2$$

and H_1 represents the applied perturbative electric field to which the linear response is to be found. Writing $\rho = \rho_0 + \sigma$, where σ is the change affected by H_1, the equation of motion for ρ becomes

$$\dot{\rho} = -i\,[H, \rho]$$

$$\dot{\sigma} = i\,[\rho_0, H_1] + i\,[\sigma, H_0]\,. \tag{2}$$

Here, only linear terms have been kept on the right hand side. Matrix elements of this equation can then be taken using the representation described in [26], which we will now briefly describe.

Given an operator Q, which in the absence of a magnetic field has the property

$$\left\langle \boldsymbol{r}|Q_{B=0}|\boldsymbol{r}'\right\rangle = f(\boldsymbol{r} - \boldsymbol{r}')$$

it is possible to show that in the presence of a magnetic field it has the property

$$\mathrm{e}^{\mathrm{i}\phi(\boldsymbol{r},\boldsymbol{r}')}\left\langle \boldsymbol{r}|Q_{B\neq 0}|\boldsymbol{r}'\right\rangle = f'(\boldsymbol{r} - \boldsymbol{r}')$$

where the Peierls factor $\mathrm{e}^{\mathrm{i}\phi(\boldsymbol{r},\boldsymbol{r}')}$ depends upon the gauge $\boldsymbol{A}$ but not on the operator Q. The function $f'(\boldsymbol{r} - \boldsymbol{r}')$ is gauge independent and this makes it natural to define a transformation

$$Q_{\boldsymbol{k}\boldsymbol{k}'} = \int \mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{r}'\,\left\langle \boldsymbol{r}|Q|\boldsymbol{r}'\right\rangle \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}\mathrm{e}^{-\mathrm{i}\boldsymbol{k}'\cdot\boldsymbol{r}'}\mathrm{e}^{\mathrm{i}\phi(\boldsymbol{r},\boldsymbol{r}')}. \tag{3}$$

Note that $Q_{\boldsymbol{k}\boldsymbol{k}'}$ is not a true matrix representation of the operator Q, and cannot be written in the form $\langle \boldsymbol{k}|Q|\boldsymbol{k}'\rangle$ where the $|\boldsymbol{k}\rangle$ are some complete set of states. Nevertheless, the resultant $Q_{\boldsymbol{k}\boldsymbol{k}'}$ values have the properties of a Wigner representation and are particularly useful here because they are gauge independent, and give rise to convenient low-field forms.

The $\boldsymbol{k}\boldsymbol{k}'$ elements of (2) are

$$\dot{\sigma}_{\boldsymbol{k}\boldsymbol{k}'} + i\,[H_0, \sigma]_{\boldsymbol{k}\boldsymbol{k}'} = i\,[\rho_0, H_1]_{\boldsymbol{k}\boldsymbol{k}'}\,. \tag{4}$$

The parts of this equation are easily evaluated. For simplicity take the gauge to be $\boldsymbol{A} = (0, -Bx, 0)$. Then $\phi(\boldsymbol{r}, \boldsymbol{r}') = \mathrm{i}(eB/2\hbar)(x + x')(y - y')$. We note that the final result will be gauge independent. Consider the two terms in the commutator $[H_0, \sigma]_{\boldsymbol{k}\boldsymbol{k}'}$ separately. Using (3) the term $(H_0, \sigma)_{\boldsymbol{k}\boldsymbol{k}'}$ reduces to

$$(H_0, \sigma)_{\boldsymbol{k}\boldsymbol{k}'} = \int \mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{r}'\,\left\langle \boldsymbol{r}|\sigma|\boldsymbol{r}'\right\rangle \mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}\mathrm{e}^{-\mathrm{i}\boldsymbol{k}'\cdot\boldsymbol{r}'}\mathrm{e}^{\mathrm{i}(eB/2\hbar)(x+x')(y-y')}$$
$$\times\frac{1}{2m}\left[\left(\hbar k_x + \frac{eB}{2}(y - y')\right)^2 + \left(\hbar k_y - \frac{eB}{2}(x - x')\right)^2\right] \tag{5}$$

where the square bracket arises from $(-i\hbar\nabla + eA)^2$ applied to $\exp(ik\cdot r + i\phi(r, r'))$. Similarly

$$(\sigma, H_0)_{kk'} = \int \mathrm{d}r\,\mathrm{d}r'\,\langle r|\sigma|r'\rangle \mathrm{e}^{ik\cdot r}\mathrm{e}^{-ik'\cdot r'}\mathrm{e}^{i(eB/2\hbar)(x+x')(y-y')}$$
$$\times\frac{1}{2m}\left[\left(\hbar k'_x - \frac{eB}{2}(y-y')\right)^2 + \left(\hbar k'_y + \frac{eB}{2}(x-x')\right)^2\right]. \tag{6}$$

Together, (5) and (6) will give the required commutator. The remaining term, $[\rho_0, H_1]_{kk'}$ can be evaluated similarly. We shall take the perturbation H_1 from the applied field to have the form

$$\langle r|H_1|r'\rangle = V_q \mathrm{e}^{iq\cdot r'}\delta(r - r').$$

This corresponds to an applied potential Φ that contains just one Fourier component. The result for $[\rho_0, H_1]_{kk'}$ is then

$$[\rho_0, H_1]_{kk'} = V_q\,(f(\epsilon_k) - f(\epsilon_{k'}))\,\delta(k + q - k'). \tag{7}$$

Here $\epsilon_k = (\hbar^2k^2/2m)$, and we have ignored the energy due to the interaction of the spin with the magnetic field. The function $f(\epsilon_k)$ is the Fermi–Dirac distribution function. Equations (5), (6) and (7) can then be inserted into the equation of motion (4). The result after changing $k \to k - q/2$ and denoting $\sigma_{k-q/2,k+q/2}$ by γ is

$$\dot\gamma + i\left(\epsilon_{k-q/2} - \epsilon_{k+q/2}\right)\gamma + i\hbar\omega_c\left(\frac{1}{i}\hat{z}\cdot k \times \frac{\partial}{\partial k}\right)\gamma = iV_q\left(f(\epsilon_{k-q/2}) - f(\epsilon_{k+q/2})\right). \tag{8}$$

This is our final expression for the equation of motion of the density operator. It is possible to generalize it to allow for band effects, as discussed in detail for the Boltzmann equation in [26]. The result is that each ϵ_k is replaced by the zero-field Bloch energy form E_k. In the next section we will consider the formal solution when there is no time dependence and calculate explicitly the response at zero temperature.

In many cases only the static response will be required and we now turn our attention to that. The static response is, for instance, suitable when one wishes to study DC conduction in a 2DEG subject to a periodic potential; it would be unsuitable when studying magnetoplasmon oscillations.

In order to simplify the expressions we will define a polar coordinate system in which the $\theta = 0$, $\phi = 0$ direction is given by the vector q. For the static case, $\dot\gamma = 0$, equation (8) then has the formal solution

$$\gamma = \mathrm{e}^{ikql_B^2\sin\theta}\int^{\theta}\frac{im_e}{\hbar^2}V_q l_B^2\,\mathrm{e}^{-ikql_B^2\sin\theta'}\left(f(\epsilon_{k-q/2}) - f(\epsilon_{k+q/2})\right)\mathrm{d}\theta' \tag{9}$$

where $l_B^2 = \frac{\hbar}{eB}$. The corresponding electron density can then be obtained from the following integral;

$$n(q) = 2\int\frac{\mathrm{d}k}{(2\pi)^2}\gamma. \tag{10}$$

The factor of two results from summation over spin variables. This integral can be performed at $T = 0$ and an explicit expression for the linear response is thus obtained.

When $T = 0$ the Fermi–Dirac distribution functions take the simple form of theta functions. Then $f(\epsilon_{k-q/2}) - f(\epsilon_{k+q/2}) = \delta f$ will be zero when k lies outside of the range

$k_f - q/2 < k < k_f + q/2$. Within this range it will take the following form.

$$\begin{aligned} &\delta f = 1 \qquad -\chi < \theta < \chi \\ &\delta f = 1 \qquad \pi - \chi < \theta < \pi + \chi \\ &\delta f = 0 \qquad \text{otherwise.} \end{aligned}$$

The angle χ is a function of k and is given by

$$\cos\chi = \left|\left(\frac{k_f^2 - k^2 - q^2/4}{kq}\right)\right|. \tag{11}$$

In addition $\int^\theta$ can be replaced by $\int_0^\theta + C_1$, where C_1 is now an integration constant. After some manipulation one can obtain the following

$$n(q) = -\frac{m_e V_q l_B^2}{\hbar^2 (2\pi)^2}\int_{k_f-q/2}^{k_f+q/2} k\,\mathrm{d}k\, 4\,\mathrm{Im}\left[\int_0^\chi \mathrm{d}\theta\, \mathrm{e}^{-\mathrm{i}\alpha\sin\theta}\int_\theta^{\pi/2}\mathrm{d}\theta'\, \mathrm{e}^{\mathrm{i}\alpha\sin\theta'}\right] + \frac{m_e V_q l_B^2}{\hbar^2(2\pi)^2}\int_{k_f-q/2}^{k_f+q/2} k\,\mathrm{d}k\int_0^{2\pi}\mathrm{d}\theta\,\mathrm{e}^{-\mathrm{i}\alpha\sin\theta} C_1. \tag{12}$$

Here $\alpha = kql_B^2$, and Im denotes the imaginary part. From (11) we can replace $k\,\mathrm{d}k$ by $\pm kq\,\mathrm{d}(\cos\chi)$. In many interesting systems, such as dot and antidot lattices, the wavelength of the applied electric field in question is long compared to the Fermi wavelength, that is $q \ll k_f$. By restricting ourselves to this case we can replace k in (12) by k_f. This letter is concerned with the response of the 2DEG in the presence of a low magnetic field. In order for $n(q)$ to remain finite as $B \to 0$ we require the constant C_1 to be zero. The integral (12) can then be evaluated by parts, and the final result is

$$n(q) = \frac{2m_e V_q}{\hbar^2\pi^2}\left[\frac{\pi}{2} - J_0(qk_f l_B^2)\right]. \tag{13}$$

Here J_0 is a Bessel function. With $V_q = -e\Phi(q)$, (13) is the linear response of the 2DEG to a long wavelength electrostatic field with potential Φ at zero temperature and low magnetic field. Notice that the Bessel function J_0 will give rise to an oscillatory response. In the limit $l_B \to \infty$ and identifying the value of the Bessel function goes to zero and (13) reduces to the linear screening for zero field. The J_0 term thus gives the correction to the zero-magnetic-field result. For low magnetic fields the argument of the Bessel function will be large. From the asymptotic form of the Bessel function we then obtain the amplitude of the correction term decreasing as $\sqrt{2/(\pi qk_f l_B^2)}$. The correction term is therefore proportional to the square root of the magnetic field. Consider now the size of the correction. A typical experimental system [11] will have an electron density of $3.0 \times 10^{15}\ \mathrm{m}^{-2}$ and an applied potential with wavelength 800 nm. For these numbers $1/\sqrt{(qk_f)} = 30.5$ nm. The correction to the screening response will then be as large as 10% for a field as low as 0.03 T. This shows that low magnetic fields will have a significant effect on the screening response of the 2DEG.

We summarize our conclusions as follows. In this letter we studied the linear screening response of a 2DEG in the presence of a low magnetic field. The response was obtained from solving the linearized equation of motion for the density operator. By making use of a representation given by [26] the calculation was greatly simplified because at no point were we required to evaluate matrix elements between Landau wavefunctions. The evaluation and summation over such matrix elements is quite often the most difficult step in other treatments [24].

At $T = 0$ the correction to the zero-field response was shown to be oscillatory in character and had an amplitude that can be quite large. In order to apply these results to systems such as the dot or antidot lattice the calculation would need to be extended to include finite temperature and a non-linear response. In practice this is very difficult. The closed form for the integration of δf was only possible at zero temperature. At a finite temperature a series expansion in powers of B can be formulated, but this would be unable to pick up a $\sqrt{B}$ dependence. However it is reasonable to suppose that the low-temperature response would be a similar size to the zero-temperature response. In certain cases, such as hopping conduction in a dot lattice [9] this would be large enough to play a significant part in the low-field magnetoresistance.

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